Transmission fluctuations and spectral rigidity of lasing states in a random amplifying medium

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(Received 5 July 1994)

Statistical properties of a random amplifying medium near the lasing threshold are considered. We show that the transmission fluctuations grow faster than the average transmission. This is related to the fluctuations of the threshold amplification value. We find that the spectrum of lasing states possesses the kind of rigidity that so far is known only for the chaotic Hamiltonian systems.

PACS number(s): 05.40.+j, 42.60.Da, 42.50.Lc

I. INTRODUCTION

The statistical approach for studying the spectral properties of a multimode quantum generator is not popular as yet. Probably, the reason is that one often considers the feedback system for an amplifying medium in the laser as an integrable resonator [1]. In a more sophisticated system, such as an amplifier integrated in an optical network that provides unavoidable random scattering or a laser resonator of nonintegrable shape, it is impossible to calculate analytically the transmission coefficients and eigenstates. From experience with the spectra of nuclei and small metallic particles [2] it is well known that this situation calls for a statistical treatment.

For a diffusive amplifying medium, Letokhov [3] using Boltzmann kinetics has calculated the lasing threshold and considered the relaxation of a lasing solution to the steady state. However, the study of interference phenomena in a random amplifying medium has not attracted much attention, despite the obvious advantage that the amplification can bring. For example, the serious obstacle in the experimental study of a phenomenon, such as Anderson localization of the light, is an absorption. This undesirable circumstance can be overcome by using a random medium that allows for pumping and therefore reverses the sign of an absorption. Weak localization in the backscattering from a medium near threshold has been addressed in the paper [4].

This paper concentrates on the statistical properties of the linear disordered multimode system, when this system approaches the lasing threshold from below. The transmission coefficient of the system is considered. Due to the disorder, it is a random quantity. Regarding the properties of mesoscopic conductors the statistics of the transmission was of considerable interest in recent years [5]. The system under consideration is an active medium that undergoes an optical phase transition. This creates a difference in the problem of mesoscopic conductors. We found that near threshold the transmission is not a self-averaging quantity, i.e., the fluctuations of the transmission coefficient become larger than the average value. Therefore, it is more appropriate to consider the fluctuations of the threshold value of the amplification. This value, which is necessary to bring a mode to lasing threshold, is a random function of the frequency. The correlation properties of this function are considered. Finally, we evaluate the fluctuation of the number of lasing solutions. We show that the spectrum possesses Dyson rigidity [6]. The interesting element here is that we find this property for a non-Hamiltonian system.

II. DESCRIPTION OF THE MODEL

Let us describe the problem under consideration in more detail. We keep in mind the propagation of electromagnetic waves (e), because the amplification is the real thing here. The direction of the polarization of electromagnetic waves relaxes in the course of only a few scattering events; therefore, we neglect the polarization and consider only a scalar version of the Maxwell equations;

$$[\Delta+k^2\varepsilon(\vec{r})]\mathrm{e}=0 \ . \tag{1}$$

Here k is the wave vector. $\varepsilon(\vec{r})=\varepsilon_1(\vec{r})+i\varepsilon_2(\vec{r})$ is a complex dielectric function. We assume that the intrinsic or artificial defects, impurities, etc., cause the fluctuation in $\varepsilon_1(\vec{r})$, so the real part of the function has a form $\varepsilon_1(\vec{r})$, $=1+\delta\varepsilon_1(\vec{r})$, where $\delta\varepsilon_1(\vec{r})$ is the white-noise-like random function with the properties $\langle\delta\varepsilon_1(\vec{r})\rangle=0$ and $\langle\delta\varepsilon_1(\vec{r}_1)\delta\varepsilon_1(\vec{r}_2)\rangle=\Lambda\delta(\vec{r}_1-\vec{r}_2)$. We also consider the approximation that allows us to introduce the dielectric function with a negative imaginary part [7]. We have in mind the system in the linear regime with the time-independent population inversion and fast dipole transitions of the pumped atoms. Due to population inversion, the imaginary part of the dielectric constant $\varepsilon_2\propto-\gamma/[(\omega-\omega_\alpha)^2+\gamma^2]$ is a negative in some frequency interval near atomic frequency ω_α of the pumped atoms. We suppose homogeneous pumping; therefore, the dependence of the imaginary part on the coordinate means that it is nonzero only in a sample. Saturation effects are neglected, and the times of relaxation of an atomic system γ^{-1} and of establishment of the population distribu-

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tion are supposed to be short compared to the time of the radiation diffusion out of the system. These assumptions allow us to neglect the dispersion of $\varepsilon_2(\vec{r})$. We intend to consider the propagation of the waves with frequencies in the region of a negative imaginary part of the dielectric function.

The important quantities that describe the propagation of the waves are the current density

$$\vec{J}=\frac{c}{2ik}\{e^*\vec{\nabla}e-e\vec{\nabla}e^*\} \tag{2}$$

and the intensity of the radiation

$$N=|e|^2 . \tag{3}$$

From Eq. (1), we have

$$\vec{\nabla}\cdot\vec{J}+ck\varepsilon_2(\vec{r})N=0 . \tag{4}$$

In the present paper, we discuss the propagation of the wave in the linear amplifying medium, where the wave undergoes multiple random scattering. We are interested in scales larger than the mean free path; therefore, the propagation of the radiation obeys the diffusion law. The diffusion coefficient is given by expression $D=cl/3$, where $l=c\tau$ is the mean free path and $1/\tau=c\Lambda k^4/4\pi$. To neglect the localization effects, we assume that the mean free path is larger than the wavelength of the radiation $lk \gg 1$.

Now we describe the geometry of the system. A schematic sample is shown in Fig. 1. The sample is of any shape in the transverse direction, and of the length L_x in the longitudinal direction. The side surface is supposed to be a perfect reflector. All sample dimensions are much larger than the mean free path.

If the flux J_0 is entering the system, then from (4) we obtain in the steady state

$$J_{\rm tr}+J_{\rm ref}-J_0=ck|\varepsilon_2|\int_V \partial\vec{r}\, N(\vec{r}) . \tag{5}$$

Here $J_{\rm tr}$ and $J_{\rm ref}$ are the total fluxes in the transmission and reflection, respectively. $V=L_xS$ is the volume of the system and S is a cross section area. The right-hand side of (5) gives the amplified value of the flux. We also introduce the transmission coefficient $J_{\rm tr}=TJ_0$ and the reflection coefficient $J_{\rm ref}=RJ_0$.

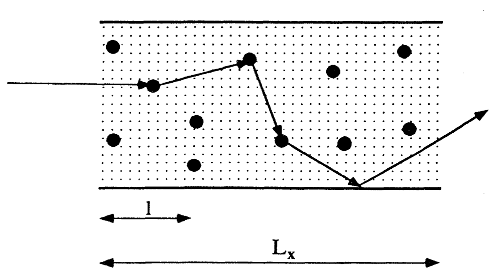

FIG. 1. Schematic picture of the system. The amplifying medium is shaded. Dots show the scattering impurities. Arrows show the path that the wave travels while undergoing the multiple scattering.

III. AVERAGE BEHAVIOR OF THE SYSTEM

Let us recall the behavior of the average intensity $\langle N(\vec{r},t)\rangle$ and, related to it, current $\langle\vec{J}(\vec{r},t)\rangle = -D\vec{\nabla}\langle N(\vec{r},t)\rangle$. The brackets denote averaging over a random realizations of $\delta\varepsilon_1(\vec{r})$.

The average intensity $\langle N(\vec{r},t)\rangle$ obeys the equation [3]

$$\frac{\partial\langle N(\vec{r},t)\rangle}{\partial t}=D\Delta\langle N(\vec{r},t)\rangle+\frac{\langle N(\vec{r},t)\rangle}{\tau_0} . \tag{6}$$

The time $1/\tau_0=ck|\varepsilon_2|$ corresponds to the amplification. Boundary conditions for (6) imply that $\langle N(\vec{r},t)\rangle$ is zero on free boundaries $x=0,L_x$, and the current throughout the insulating surface is zero. For a discussion of the boundary conditions, see Ref. [8].

Considering the relaxation of some perturbation at a large time, we have

$$\langle N(\vec{r},t)\rangle \propto \exp\left\{-t\left[D\left[\frac{\pi}{L_x}\right]^2-\frac{1}{\tau_0}\right]\right\} . \tag{7}$$

Here the first term on the right-hand side of the equation describes a diffusive decay of the intensity. The second term describes the amplification effect. If the size of the system is larger than the critical one, the second term wins, and any fluctuation of the intensity become unstable. For the critical size the growth of the intensity is balanced by the current outgoing from the boundaries, and the relaxation time goes to infinity.

It is convenient to measure the length of the sample in the units of the critical size

$$L_x \equiv L_{\rm cr}(1-\Delta) , \quad \text{where } L_{\rm cr}\equiv\pi\sqrt{D\tau_0} . \tag{8}$$

We also need a solution of (6) for the steady state with the given external flux. Solving (6) with sources of the radiation distributed on a free surface $x=0$ [8], we obtain

$$\langle N(\vec{r})\rangle = N_0\frac{\sin\left[\pi\dfrac{L_x-x}{L_{\rm cr}}\right]}{\sin\left[\pi\dfrac{L_x}{L_{\rm cr}}\right]} \approx \frac{N_0}{\pi\Delta}\sin\left\{\pi\left[1-\Delta-\frac{x}{L_{\rm cr}}\right]\right\} . \tag{9a}$$

Here the second equality is valid in the limit $\Delta\to 0$. Taking the derivative of (9a), we obtain an expression for the current

$$\langle J^x(\vec{r})\rangle=\frac{DN_0}{L_{\rm cr}\Delta}\cos\left\{\pi\left[1-\Delta-\frac{x}{L_{\rm cr}}\right]\right\} . \tag{9b}$$

Here we introduce quantity $N_0=3|n\cdot J_0|/c$. $\vec{n}$ and $\vec{J}_0$ are normal to the surface unit vector and the incident flux, respectively. Calculating (9b) on the boundary $x=L_x$, we obtain expression for the average transmission coefficient

$$\langle T\rangle=\frac{\langle J^x(x=L_x)\rangle}{|\vec{n}\cdot J_0|}=\frac{l}{L_{\rm cr}\Delta} . \tag{10}$$

This expression contains the additional amplification factor Δ^{-1} compared to the transmission coefficient of an elastic medium. Let us mention that in the reflection we also have amplifying $\langle R \rangle = 1 + l/L_{cr}\Delta$. To obtain this formula for the reflection coefficient, we use definitions (5) and (9a) for the mean value of the intensity $\langle N(\vec{r}) \rangle$.

The solution of the intensity in the critical state one can obtain from (9a) in the limit Δ, $N_0 \to 0$, but keeping finite Δ/N_0. Saying it backward, the critical size can be determined as a size of the system that sustains the finite intensity of the radiation in the limit of zero external flux. In the next section we will apply this fact for studying the fluctuation of the threshold.

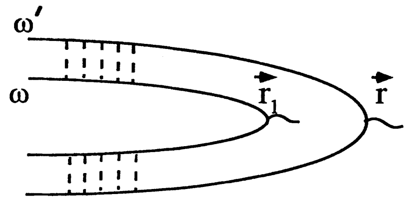

FIG. 2. Diagram describing the correlation function of the Langevin currents at frequencies ω and ω' in the points $\vec{r}$ and $\vec{r}_1$, respectively. Solid lines describe the Green's function of Eq. (1), while dashed lines describe the scattering of the waves.

IV. TRANSMISSION AND FLUCTUATIONS OF THE THRESHOLD

We will see shortly that the statistical fluctuations become important near the threshold. To consider them, we will apply the Langevin approach that initially was developed in the study of the fluctuations in an elastic random medium [9]. Besides the average part, the current contains the random contribution that can be expressed as

$$\delta\vec{J}(\vec{r}) = -D\vec{\nabla}\delta N(\vec{r}) + \vec{J}_{ex}(\vec{r}) \ . \tag{11}$$

Here $\vec{J}_{ex}(\vec{r})$ is the Langevin random current. In the case of the coherent monochromatic incident wave correlation function of the Langevin current is given by expression [9].

$$\langle J^i_{ex}(\vec{r}) J^k_{ex}(\vec{r}') \rangle = \frac{\lambda^2 c^2 l}{6\pi} \langle N(\vec{r}) \rangle^2 \delta_{ik} \delta(\vec{r}-\vec{r}') \ . \tag{12a}$$

This expression can be derived by summing the diagrams shown in Fig. 2. Expression (12a) describes the Langevin currents of the radiation of the same frequency averaged over the scale of the mean free path. Correlation of the Langevin currents of the monochromatic waves of the different frequencies ω and ω' in the limit $\Delta \to 0$ is given by the expression

$$\langle J^i_{ex}(\vec{r},\omega) J^k_{ex}(\vec{r}',\omega') \rangle = \frac{\lambda^2 c^2 l}{6\pi} \frac{\Delta^2}{\Delta^2 + [\tau_0(\omega-\omega')]^2} \langle N(\vec{r}) \rangle^2 \delta_{ik} \delta(\vec{r}-\vec{r}') \ . \tag{12b}$$

Conservation law (4) reads that

$$\vec{\nabla}\delta \cdot \vec{J}(\vec{r}) = \frac{\delta N(\vec{r})}{\tau_0} \ . \tag{13}$$

Combining (11) and (13), for the fluctuations of the intensity $\delta N(\vec{r})$, we obtain the following equation:

$$-D\Delta\delta N(\vec{r}) + \vec{\nabla} \cdot \vec{J}_{ex}(\vec{r}) = \frac{\delta N(\vec{r})}{\tau_0} \ . \tag{14}$$

Equation (14) is supplemented by the same boundary condition as (6), $\delta N(\vec{r}) = 0$ at free boundaries $x = 0, L_x$, while the fluctuating current throughout the insulating surface is zero.

Let us reexamine the problem of the transmission near the threshold. Sample-specific transmission differs from the average (10) by the amount

$$\delta T \equiv T - \langle T \rangle = |\vec{J}_0 \cdot \vec{n}|^{-1} \int_{x=L_x} \partial\vec{S}\delta\vec{J}(\vec{r}) \ .$$

Averaging the square of it, we obtain the following expression:

$$\langle (\delta T)^2 \rangle = D^2 \int_{x=x'=L_x} \delta S_x \delta S'_x \frac{\partial}{\partial x} \frac{\partial}{\partial x'} \int_V \partial\vec{r}_1 \partial\vec{r}_2 \left[\frac{\partial}{\partial x_1} G(\vec{r};\vec{r}_1;0) \frac{\partial}{\partial x_2} G(\vec{r}';\vec{r}_2;0) \right] \langle J^x_{ex}(\vec{r}_1) J^x_{ex}(\vec{r}_2) \rangle \ . \tag{15}$$

Deriving (15), we take into account that according to (12a) the Langevin currents are zero on the free boundaries. Here $G(\vec{r};\vec{r}_1;\omega)$ is the Green's function of the diffusion equation (6) at frequency ω. We need this function integrated over the cross-section area for small Δ,

$$\int \partial S_x G(\vec{r};\vec{r}_1;\omega) = \frac{L_{cr} \left\{ \cos\pi \left[1 - \frac{(x+x')}{L_{cr}} \right] - \cos\pi \left[1 - \frac{|x-x'|}{L_{cr}} \right] \right\}}{\pi^2 D(\Delta + i\omega\tau_0)} \ . \tag{16}$$

Using this expression and (9a), we obtain

$$\frac{\langle(\delta T)^2\rangle}{\langle T\rangle^2}=\frac{L_{\rm cr}}{8\pi\Delta^2\xi_{\rm loc}} \ . \tag{17}$$

Here we introduce $\xi_{\rm loc}\equiv 2\pi dS/3\lambda^2$, which is the localization length in a sample without the amplification [10]. We assume that it is much larger than the length of the sample L_x, so we can ($L_x \ll \xi_{\rm loc}$) neglect the localization effects on the transmission.

Fluctuations near the threshold grow faster than the average, therefore T is not the self-averaging quantity. This situation can be interpreted in terms of the fluctuations of the threshold value of the amplification. It is clear from the above that the threshold corresponds to the situation when the nonzero density of the radiation exists in the limit of zero external flux. For the average intensity, this happens when $\Delta=0$. In the general case, we need to find this condition for Eq. (1). Because of the random nature of (1) instead of trying to solve it, we turn to the calculation of fluctuations of the threshold. To do this we consider the threshold value of τ_0^{-1}, for which $\langle N(\vec r)\rangle+\delta N(\vec r)$ has the nonzero value in the limit of zero external flux. Let us consider the quantity $\vec\nabla\vec J_{\rm ex}(\vec r)$ in the equation for $\langle N(\vec r)\rangle+\delta N(\vec r)$ [it is the same as (14)] as a small perturbation. Perturbation theory gives the first order correction to the average threshold

$$\tilde\Omega=\lim_{\Delta\to 0}\frac{\int_V \partial\vec r\langle N(\vec r)\rangle^2\vec\nabla\cdot\vec J_{\rm ex}(\vec r)}{\int_V\partial\vec r\langle N(\vec r)\rangle^2} \ . \tag{18}$$

This correction depends on the realization of the impurity configuration, and to estimate its value we calculate $\langle\tilde\Omega^2\rangle$. Using expression (12a) for the correlation function of the Langevin currents near the threshold, we obtain

$$\langle\tilde\Omega^2\rangle=\tau_0^{-2}\frac{8L_{\rm cr}}{\pi^2\xi_{\rm loc}} \ . \tag{19}$$

One easily can see that if the transmission has the power law divergency near the corrected threshold $T\propto l/(\Delta+\tilde\Delta)L_{\rm cr}$, where $\tilde\Delta\propto\tau_0^{-1}\tilde\Omega$ is a random correction to the threshold, then making use of (19) we obtain the expression (17).

Equation (18) does not specify the frequency dependence of the threshold. It is clear from the nature of the problem that the threshold is a random function of the frequency. Let us consider the correlation function $\langle\tilde\Omega(\omega)\tilde\Omega(\omega')\rangle$. If we assume that expression (18) determines $\tilde\Omega$ for some frequency, then using (12b) we find the correlation function as the limit $\Delta\to 0$ of an expression

$$\langle\tilde\Omega(\omega)\tilde\Omega(\omega')\rangle=\tau_0^{-2}\frac{8L_{\rm cr}}{\pi^2\xi_{\rm loc}}\frac{4\Delta^2\tau_0^{-2}}{4\Delta^2\tau_0^{-2}+(\omega-\omega')^2} \ .$$

It is reasonable that in this correlation function the minimum value of $\Delta^2\tau_0^{-2}$ must be of the order of the fluctuation (19) of the threshold itself. This condition determines a correlation scale of the frequencies as $\propto\tau_0^{-1}\sqrt{(L_{\rm cr}/\xi_{\rm loc})}$.

The outcome of these results might be as follows. It is clear from the physical meaning of $\langle\tilde Q^2\rangle$ that expression (19) gives the estimation of the interval of the amplification; one needs to generate all modes. We also may conclude that correlation in appearance of the modes at a distance larger than $\tau_0^{-1}\sqrt{L_{\rm cr}/\xi_{\rm loc}}$ from each other is suppressed.

V. FLUCTUATION OF THE NUMBER OF LASING SOLUTIONS

When the system is close to the lasing threshold, a discrete structure of modes appears. Standard expressions [1,6] for the mode incorporate shift of the eigenfrequencies of resonator (which are precursors of the lasing modes) due to the dispersion of the real and imaginary parts of the dielectric function (energy pulling). In the system under consideration, one might expect that "resonances" of the cavity are broadened by the order of π^2D/L_c^2 due to the diffusive escape from the system, and mode spacing is of the order of $\rho=1/V\nu$, where ν is the density of states in the region of interest. The ratio of spacing to the broadening is of the order of $L_x/\xi_{\rm loc}\ll 1$. Near threshold, when pumping compensates for the escape rate $\pi^2D/L_x^2\approx\tau_0^{-1}$, the modes become well developed. Our system is a random one; therefore the statistic of the modes is of importance. On average, there are $\langle N\rangle=\delta\omega/\rho$ modes in the frequency interval $\delta\omega$. We are speaking of the intervals smaller than the atomic broadening near the center of atomic frequency, where the amplification is maximum, and therefore neglect the dispersion of the dielectric function.

For the fluctuation of the number of modes, our result is

$$\langle(\delta N)^2\rangle=2\pi^{-2}\ln\langle N\rangle \ . \tag{20}$$

Here $\delta N=N-\langle N\rangle$. It means that the rigidity of the spectrum exists as it is for the Hamiltonian systems [6]. Formula (20) coincides with the formula for the orthogonal Wigner-Dyson ensemble of real random matrixes. This is surprising in view of the fact that our system is dissipative and its evolution is described by a non-hermitian equation.

Let us briefly describe the derivation of expression (20). Usually the number of energy levels is given by

$$N=\int_{\delta\omega}\partial\omega\,\nu_V(\omega) \ . \tag{21}$$

In expression (21), the density of states in a sample is determined as

$$\nu_V(\omega)=\int_V\partial\vec r\sum_n\delta(\omega-\omega_n)|\Psi_n(\vec r)|^2 \ ,$$

$$\int_V\partial\vec r|\Psi_n(\vec r)|^2=1 \ . \tag{22}$$

Here ω_n and $\Psi_n(\vec r)$ are the eigenvalues and eigenstates, respectively. Also, we can rewrite (22) in terms of the retarded and advanced Green's functions:

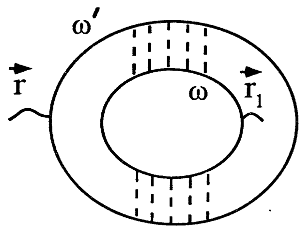

FIG. 3. Diagram describing the correlation function of local densities of states at frequencies ω and ω' in the points $\vec{r}$ and $\vec{r}_1$, respectively. To obtain the correlation function of the density of the states, one must integrate over $\vec{r}$ and $\vec{r}_1$.

$$\nu_V(\omega)=(2\pi)^{-1}\int_V \partial\vec{r}\sum_n\left[\frac{1}{\omega-\omega_n-i\delta}-\frac{1}{\omega-\omega_n+i\delta}\right]|\Psi_n(\vec{r})|^2=(2\pi)^{-1}\int_V \partial\vec{r}[G_\omega^R(\vec{r},\vec{r})-G_\omega^A(\vec{r},\vec{r})] . \quad (23)$$

At this point, we need to make a comment. We calculate the density of states through the Green's function of Eq. (1). Definitions (22) and (23) come from the spectral representation of the Green's function of a finite system. In our case, however, $\Psi_n(\vec{r})$ are extended eigenstates; those asymptotics correspond to outgoing waves. Therefore, we cannot rigorously prove (21)–(23), but we can check them for a solvable case. We did it for a one-dimensional case with only two mirrors (the Fabry-Pérot-type resonator). One can check that the correction to (23) in this case is of the order of λ/L_x; therefore, we assume that (21), (22), and (23) hold up to a possible small correction.

Taking the average of two Green's functions as is shown in Fig. 3, we obtain an expression:

$$\langle\nu_V(\omega)\nu_V(\omega')\rangle=\pi^{-2}\mathrm{Re}\int_V \partial\vec{r}_1\partial\vec{r}_1 G(\vec{r}_1;\vec{r}_2;\omega-\omega')\times G(\vec{r}_2;\vec{r}_1;\omega-\omega') . \quad (24)$$

More details about this kind of averaging can be found in [9] and [11]. In expression (24), $G(\vec{r}_1;\vec{r}_2;\omega-\omega')$ is the Green's function of the diffusion equation (6). Calculating it in the limit $\Delta\to 0$, we obtain

$$\langle\nu_V(\omega)\nu_V(\omega')\rangle=\pi^{-2}\mathrm{Re}\left[\frac{2\Delta}{\tau_0}+i(\omega-\omega')\right]^{-2} . \quad (25)$$

Taking the integrals of (25) over frequencies, we arrive at (20).

VI. CONCLUSION

We have considered some statistical properties of the linear random amplifying medium. It is shown that fluctuations in the transmission diverge faster than the average near the threshold. This is a consequence of the fluctuations of the threshold value of the amplification. Threshold is also a random function of frequency, with the variance and correlation scale increasing with rising disorder. The fluctuation of the number of lasing solutions shows a rigidity similar to that of the Hamiltonian system. We should stress that our system is not one of Wigner-Dyson classification, and the distribution of modes must also describe the distribution of their imaginary parts. Based on the physical background, their variance is proportional to that of the threshold.

So far we considered a medium that is far from the Anderson transition ($L_x \ll \xi_{\mathrm{loc}}$). In the localization regime, when $L_x \geq \xi_{\mathrm{loc}}$, on a physical basis we may expect that the picture of generation must change drastically, the threshold decreases with increasing L_x exponentially, and the fluctuations of the threshold become very large. It is also of interest to consider the influence of the amplification on the localization. In [4] it was shown that the localization effects in the backscattering are strongly enhanced near the threshold. We will consider the questions of interplay between the localization and generation in a future work.

ACKNOWLEDGMENTS

I thank N. Rosengaard, R. Serota, and K. Shestak for helpful conversation. This work was supported by the Russian Academy of Sciences Grant No. 93-02-02567, and in part by the U.S. Army, Grant No. DAAL03-93-G-0077. The hospitality of the Department of Physics, University of Cincinnati, is gratefully acknowledged.

[1] O. Svelto, *Principles of Lasers,* edited by D. C. Hanna (Plenum, New York, 1989).

[2] T. A. Brody, J. Flores, J. Bruce French, P. A. Mello, A. Pandey, and Samuel S. M. Wong, Rev. Mod. Phys. **53**, 385 (1981).

[3] V. S. Letokhov, Zh. Eksp. Teor. Fiz. **53**, 1442 (1967) [Sov. Phys. JETP **26**, 835 (1968)].

[4] A. Yu. Zyuzin, Europhys. Lett. **26**, 517 (1994).

[5] A. D. Stone, P. A. Mello, K. A. Muttalib, and J.-L. Pichard, in *Mesoscopic Phenomena in Solids,* edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991).

[6] F. J. Dyson, J. Math. Phys. **3**, 166 (1962); F. J. Dyson and M. L. Mehta, *ibid.* **4**, 701 (1963).

[7] K. Shimoda, *Introduction to Laser Physics* (Springer-Verlag, Berlin, 1984).

[8] M. J. Stephen, in *Mesoscopic Phenomena in Solids* (Ref. [5]).

[9] A. Yu. Zyuzin and B. Z. Spivak, Zh. Eksp. Teor. Fiz. **93**, 994 (1987). [Sov. Phys. JETP **66**, 560 (1987)].

[10] K. B. Efetov, Adv. Phys. **32**, 53 (1983).

[11] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].